Strategy Logic: Origin, Results, & Open Questions

Fabio Mogavero

Università degli Studi di Napoli Federico II

Vardi Fest '22

August 1st, 2022

Origin of SL

* Everything began at the end of August '08 while visiting Moshe

Origin of SL

* EVERYTHING BEGAN AT THE END OF August '08 WHILE VISITING Moshe

Just learned two weeks before at ESSLLI '08

• Study "ATL* extended with memoriful temporal quantifications

only heard about it

Origin of SL

* EVERYTHING BEGAN AT THE END OF AUGUST '08 WHILE VISITING MOSHE

Just learned two weeks before at ESSLLI '08

- Study ATL^* extended with **memoriful temporal quantifications**

only heard about it

The idea was to formalize the notion of **Relentful Strategic Reasoning**

$$s_0 \bullet \xrightarrow{\pi_1}_{\pi_2} \bullet \; s_1 \models \langle\langle A \rangle\rangle \psi$$
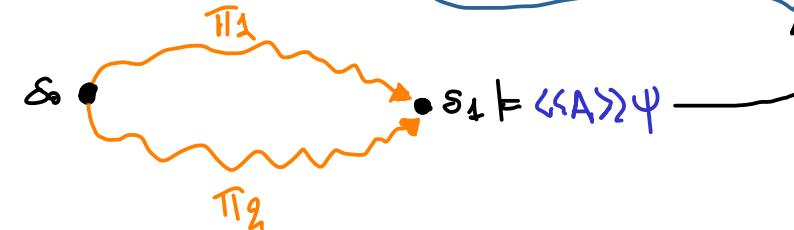
Origin of SL

3/4 - Weeks

* A Moshe's Trip and one Hurrican Later...

Hurrikan Ike
my 1st and fortunately only hurricane
11 days with no electricity at home

Origin of SL

3/4 - Week 6

* A MOSHE's TRIP AND ONE HURRICAN LATER...

- Positive aspect

  - there was no problematic interaction between strategy quantification and memoryful interpretation of time

Origin of SL

3/4 - Week 6

* A Moshe's Trip and one Hurrican Later...

- Positive aspect

  - there was no problematic interaction between strategy quantification and memoryful interpretation of time

- Negative aspects

  no definition or proof

  - the decision procedures for ATL* relied on an implicit assumption now known as behavioral quantification

Origin of SL

3/4 - Week6

* A Moshe's trip and one Hurrican later...

- Positive aspect

  - there was no problematic interaction between strategy quantification and memoryful interpretation of time

- Negative aspects

  no definition or proof

 - the decision procedures for ATL* relied on an implicit assumption now known as behavioral quantification

  - the proof of the bounded-model property of ATL* was indirect and ad-hoc

  crucial for the decidability of the satisfiability problem

Origin of SL

Behavioral Strategy Quantifications
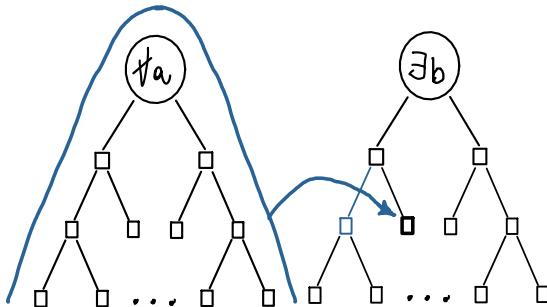
$$Ag = \{a, b\} \; ; \; [[a_2]] \, \psi$$


Non-Behavioral Quantification

Behavioral Quantification

Quantifications over Strategies can be Simulated by
Recurrent Quantifications over Actions

Origin of SL

Behavioral Strategy Quantifications
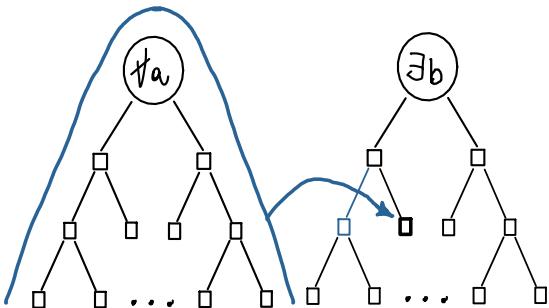
$$Ag = \{a, b\} \; ; \; [\![a]\!] \, \psi$$

Non-Behavioral Quantification   Behavioral Quantification

Quantifications over Strategies can be Simulated by Recurrent Quantifications over Actions
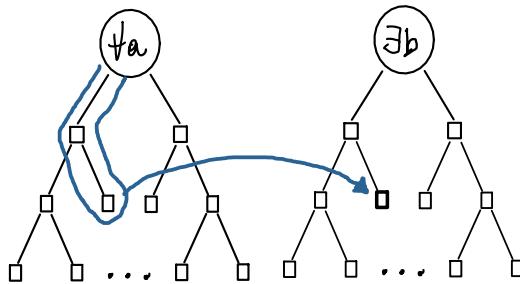
Bounded-Model Property
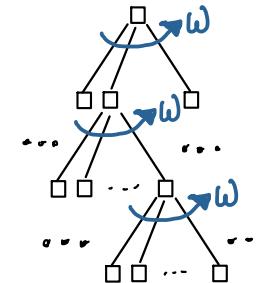
φ ATL* FORMULA

Unbounded Tree Model   K-Bounded Tree Model
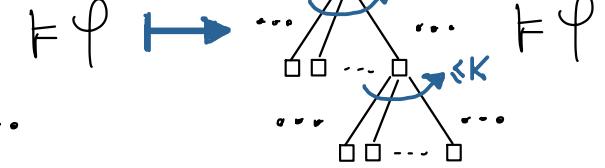
Only Finitely Many Actions are Required to Satisfy an ATL* Formula

Origin of SL
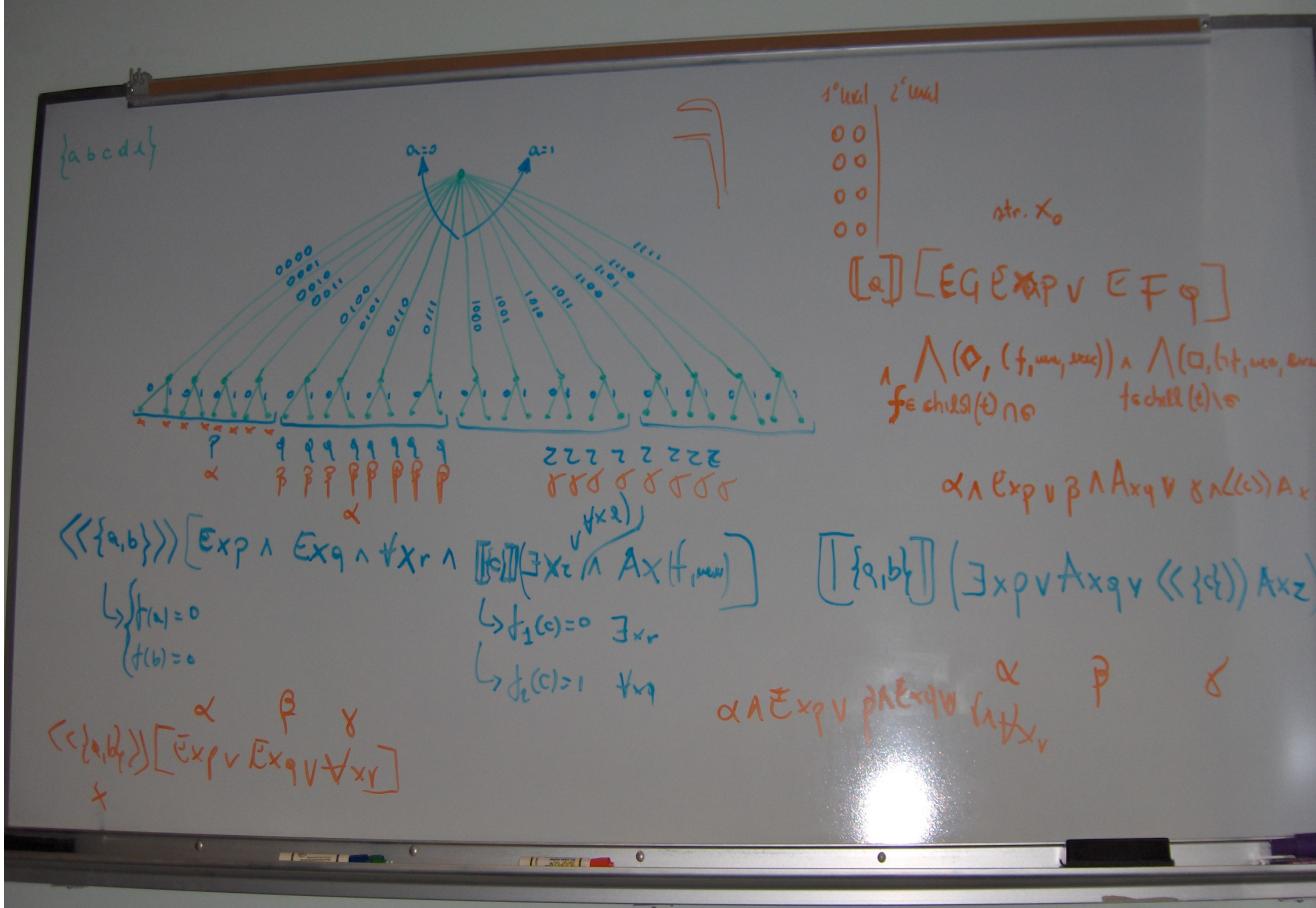
WHITE BOARD IN MY OFFICE AT THE RICE UNIVERSITY
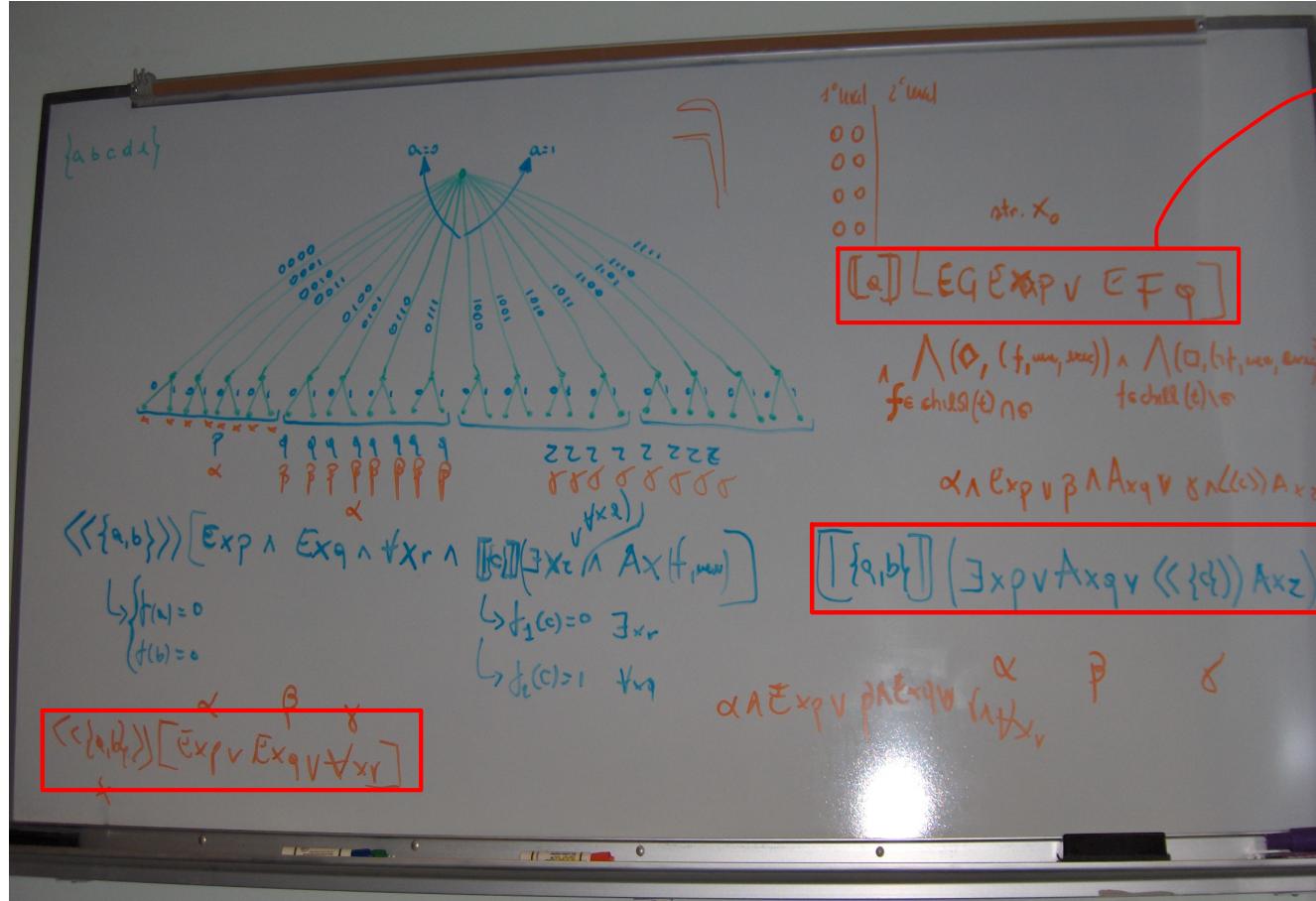
Origin of SL

Original Ugly Syntax

White Board in my office at the Rice University
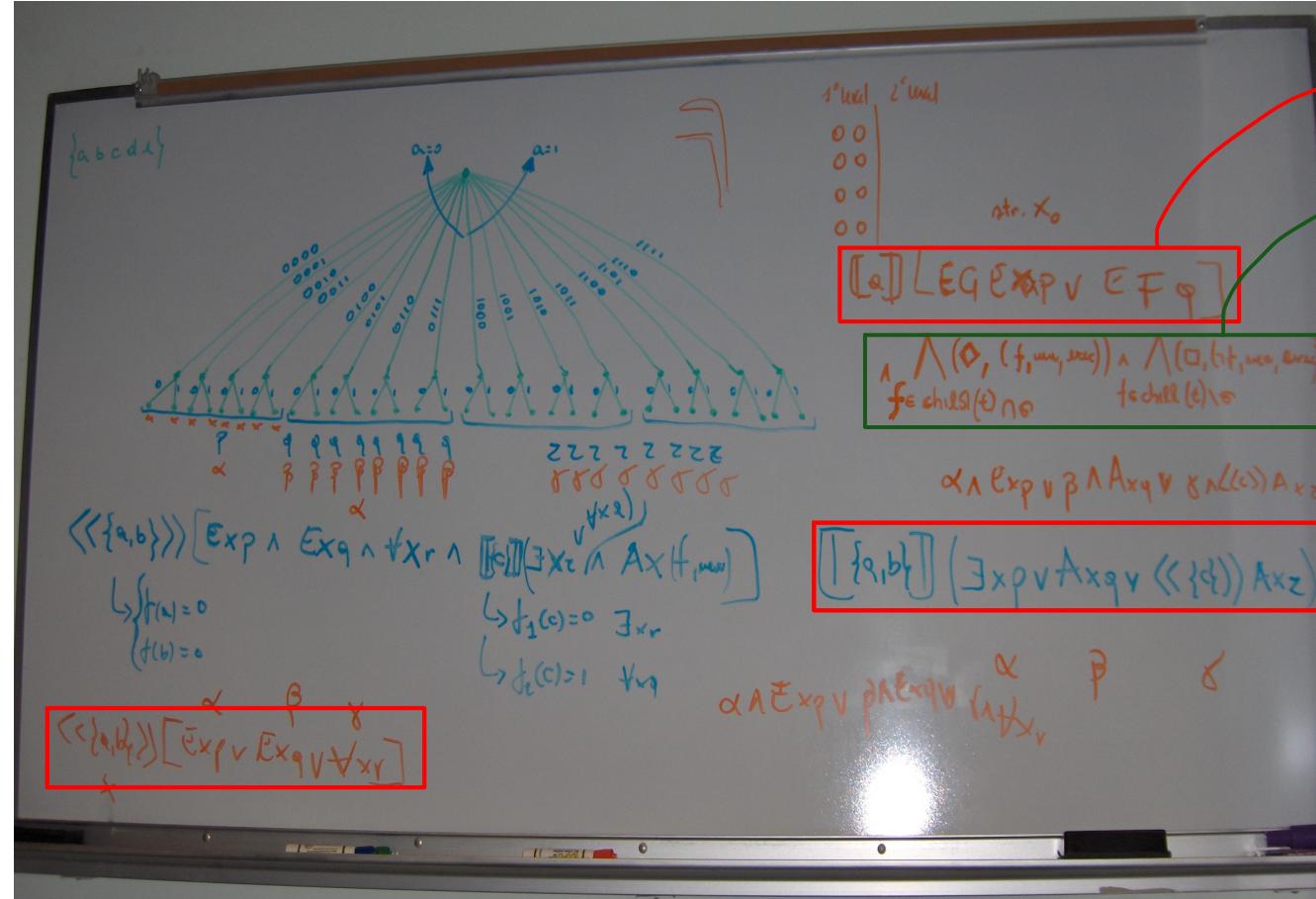
Origin of SL

White Board in my office at the Rice University
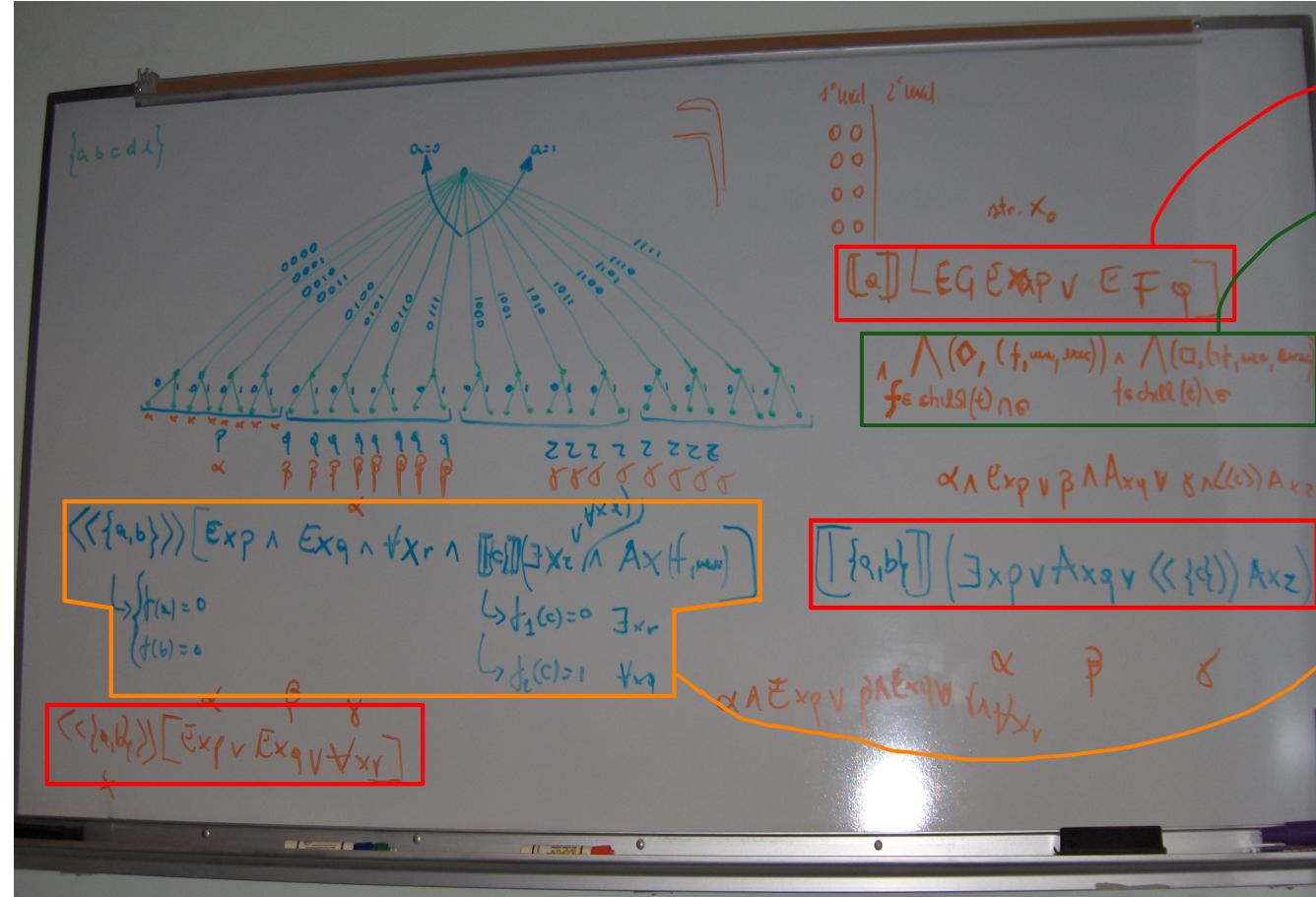
Origin of SL

Original Ugly Syntax

Tentative encoding into Symmetric Tree Automata

Attempt of Falsifying the Bounded-Model Property

White Board In my office at the Rice University

Origin of SL

Rays of Aton

Ancient Egyptians Studying Modal Logics
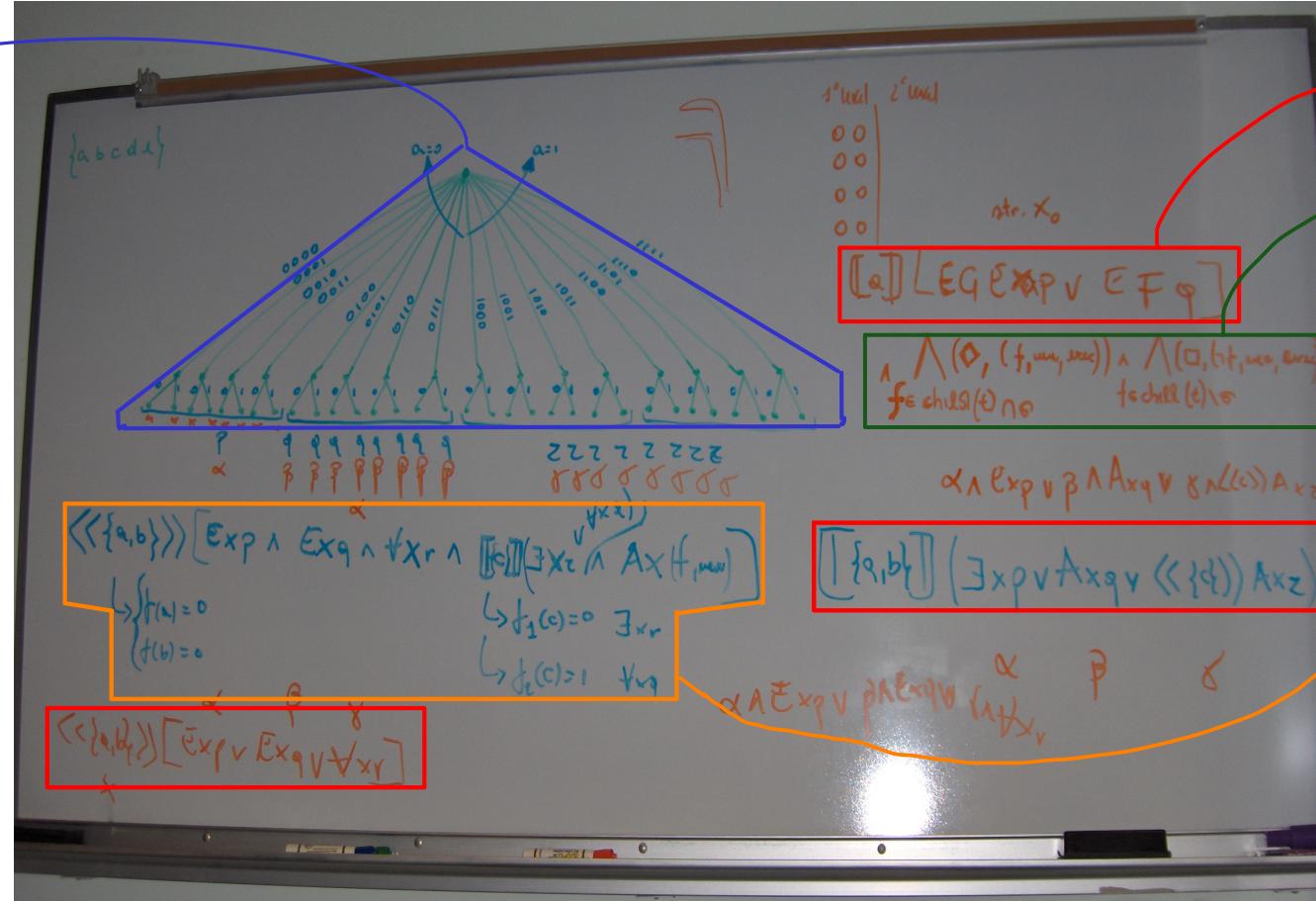
Original Ugly Syntax

Tentative encoding into Symmetric Tree Automata

Attempt of Falsifying the Bounded-Model Property

White Board In My office At the Rice University

Origin of SL

* §In the Beginning was Multi-Player Interaction Logic ...

  • Messy syntax and complex grammar: more similar to ATL* with strategy contexts than to SL

Origin of SL

* § IN THE BEGINNING WAS MULTI-PLAYER INTERACTION LOGIC ...

  • Messy syntax and complex grammar: more similar to ATL* with strategy contexts than to SL

* DURING ONE OF OUR WEEKLY MEETINGS, MOSHE SAID: "WE HAVE TO SIMPLIFY!"

  complete decoupling of quantifications & bindings (as in FOL)

  • Strategy Logic was born: $\varphi := p \mid \neg\varphi \mid \varphi \wedge \varphi \mid \varphi \vee \varphi \mid X\varphi \mid \varphi U \varphi \mid \varphi R \varphi \mid \langle\langle x \rangle\rangle \varphi \mid [\![x]\!] \varphi \mid (a, x) \varphi$

  LTL   STRATEGY   AGENT-STRATEGY
             QUANTIFIERS   BINDING

  └ Originally in post-fix notation $\varphi(a, x)$: reviewers did not like it!

Origin of SL

* $\S$ In the Beginning was Multi-Player Interaction Logic ...

  • Messy syntax and complex grammar: more similar to ATL* with strategy contexts than to SL

* During one of our Weekly meetings, Moshe said: "We have to simplify!"

  • Strategy Logic was born: $\varphi := p \mid \neg\varphi \mid \varphi \wedge \varphi \mid \varphi \vee \varphi \mid X\varphi \mid \varphi U\varphi \mid \varphi R\varphi \mid \langle\langle x \rangle\rangle \varphi \mid [[x]]\varphi \mid (a,x)\varphi$

LTL   STRATEGY QUANTIFIERS   AGENT-STRATEGY BINDING

complete decoupling of quantifications & bindings (as in FOL)

⌐ Originally in post-fix notation $\varphi(a,x)$: reviewers did not like it!

But the name came later: SL is a multi-agent concurrent extension of Strategy Logic by Chatterjee, Henzinger, & Piterman '07
Moshe said: "This seems to be the Ultimate Strategy Logic"

Origin of SL

* My visit at Rice University ended in December '08...

  - Still e (metric) ton of questions, but also few promising ideas

I saw the snow in Houston!
A week later, it was summer again

Origin of SL

* My visit at Rice University ended in December '08...

  - Still a (metric) ton of questions, but also few promising ideas

* Moshe conjectured the decidability of the satisfiability problem for SL

  - It took me 10 months to disprove that conjecture! ◄

Origin of SL

* My visit at Rice University ended in December '08...

  - Still a (metric) ton of questions, but also few promising ideas

* Moshe conjectured the decidability of the satisfiability problem for SL

  - It took me 10 months to disprove that conjecture !

  - We then started analysing several fragments; one of these proved to be decidable

    extends ATL*

This gave me the idea of studying similar fragments of FOL, starting a new classification project

Origin of SL

* MY VISIT AT RICE UNIVERSITY ENDED IN DECEMBER '08 ...

  - Still a (metric) ton of questions, but also few promising ideas

* MOSHE CONJECTURED THE DECIDABILITY OF THE SATISFIABILITY PROBLEM FOR SL

  - It took me 10 months to disprove that conjecture!

  - We then started analysing several fragments; one of these proved to be decidable

    extends ATL*

  This gave me the idea of
  studying similar fragments of FoL, starting a new classification project

* THE STORY ABOUT THE BEHAVIORAL PROPERTY IS LEFT FOR ANOTHER TIME

I saw the snow in Houston!
A week later, it was summer again

I showed my results to
Moshe, when I visited him
& Orna at Hebrew Uni
in February '10

Results on SL

* (Boolean-Goal) Strategy Logic (SL[BG])

  - Does not satisfy the behavioral-quantification property

  - The model-checking problem is non-elementary-"e"

  - Does not enjoy the bounded-model property

  - The satisfiability problem is higly undecidable

Results on SL

* **(Boolean-Goal) Strategy Logic (SL[BG])**
  * Does not satisfy the behavioral-quantification property
  * The model-checking problem is non-elementary-"c"
  * Does not enjoy the bounded-model property
  * The satisfiability problem is highly undecidable

* **One-Goal Strategy Logic (SL[1G])**
  * Does satisfy the behavioral-quantification property
  * The model-checking problem is 2ExpTime-c
  * Does enjoy the bounded-model property
  * The satisfiability problem is 2ExpTime-c

Results on SL

* **(Boolean-Goal) Strategy Logic (SL[BG])**
  - Does not satisfy the behavioral-quantification property
  - The model-checking problem is non-elementary-"c"
  - Does not enjoy the bounded-model property
  - The satisfiability problem is highly undecidable

* **One-Goal Strategy Logic (SL[1G])**
  - Does satisfy the behavioral-quantification property
  - The model-checking problem is $2\text{ExpTime}-c$
  - Does enjoy the bounded-model property
  - The satisfiability problem is $2\text{ExpTime}-c$

* **Conjunctive/Disjunctive/Alternating/E-Goal Strategy Logic (SL[C/D/A/E])**
  - Do satisfy a kind of behavioral-quantification property
  - The model-checking problem is $2\text{ExpTime}-c$
  - Only the Flat fragment of SL[CG] is known to $\begin{cases} \text{enjoy the bounded-model property} \\ \text{have a } 2\text{ExpTime}-c \text{ satisfiability problem} \end{cases}$

Results on SL

* **(Boolean-Goal) Strategy Logic (SL[BG])**
  * Does not satisfy the behavioral-quantification property
  * The model-checking problem is non-elementary-"c"
  * Does not enjoy the bounded-model property
  * The satisfiability problem is highly undecidable

* **One-Goal Strategy Logic (SL[1G])**
  * Does satisfy the behavioral-quantification property
  * The model-checking problem is 2ExpTime-c
  * Does enjoy the bounded-model property
  * The satisfiability problem is 2ExpTime-c

* **Conjunctive/Disjunctive/Alternating/E-Goal Strategy Logic (SL[C/D/A/E])**
  * Do satisfy a kind of behavioral-quantification property
  * The model-checking problem is 2ExpTime-c
  * Only the Flat fragment of SL[CG] is known to {enjoy the bounded-model property, have a 2ExpTime-c satisfiability problem

DO NOT FORGET
MOSHE'S MOTTO!

2ExpTime-c is the new PTime

:)

Open Questions

via Alternating Hodges' Semantics (for QPTL, to appear in ToCL)

* Unifying Compositional Behavioural Semantics

• Would this always ensure **elementary complexity**?

↳ for the decidable problems

Open Questions

via Alternating Hodges' Semantics (for QPTL, to appear in ToCL)

* UNIFYING COMPOSITIONAL BEHAVIOURAL SEMANTICS

• Would this always ensure **elementary complexity**?

↳ for the decidable problems

* ARE THERE OTHER SEMANTICS WITH ELEMENTARY COMPLEXITY?

↳ some candidates are those used in delay games

Open Questions

via Alternating Hodges' Semantics (for QPTL, to appear in TOCL)

* Unifying Compositional Behavioural Semantics

* Decidability of Fragments of Strategy Logic

- Would this always ensure **elementary complexity**?

  ↳ for the decidable problems

  - Full Conjunctive-Goal
  - (Flat/Full) Disjunctive/Alternating/E-Goal

* Are There Other Semantics with Elementary Complexity?

  ↳ some candidates are those used in delay games

Open Questions

via Alternating Hodges' Semantics (for QPTL, to appear in ToCL)

* Unifying **Compositional Behavioural Semantics**

- Would this always ensure **elementary complexity**?

  ↳ for the decidable problems

* Are There **Other Semantics** with Elementary Complexity?

  ↳ Some candidates are those used in delay games

* **Decidability** of Fragments of Strategy Logic

- Full Conjunctive-Goal
- (Flat/Full) Disjunctive/Alternating/E-Goal

* Back and Forth from **First-Order Logic**

- Do decidable fragments of **FOL** always lead to decidable fragments of **SL** and vice versa?

~o~o~o~ Thank You Very Much Moshe ~o~o~o~